9.1.2 DEFINITION OF DERIVATIVE AND DIFFERENTIABILITY

Let $f(x)$ be a function defined on an open interval containing the point 'a'. If

$\lim_{\delta x \to 0}\left(\frac{f(a+\delta x)-f(a)}{\delta x}\right)$ exists, then f is said to be differentiable at $x=a$ and this limit is said to be the derivative of f at a and is denoted by $f'(a)$.

We can calculate derivative of 'f' at any point x in the domain of f.

Let $y=f(x)$ be a function. Let there be a small increment in the value of 'x', say δx, then correspondingly there will be a small increment in the value of y say δy.

$\therefore\ y+\delta y = f(x+\delta x)$

$\therefore\ \delta y = f(x+\delta x) - y$

$\delta y = f(x+\delta x)-f(x)$ .... $[\because y=f(x)]$

As δx is a small increment and $\delta x \neq 0$, so dividing throughout by δx, we get $\frac{\delta y}{\delta x} = \frac{f(x+\delta x)-f(x)}{\delta x}$

Now, taking the limit as $\delta x \to 0$ we get

$$\lim_{\delta x\to 0}\left(\frac{\delta y}{\delta x}\right) = \lim_{\delta x \to 0}\left(\frac{f(x+\delta x)-f(x)}{\delta x}\right)$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

☆

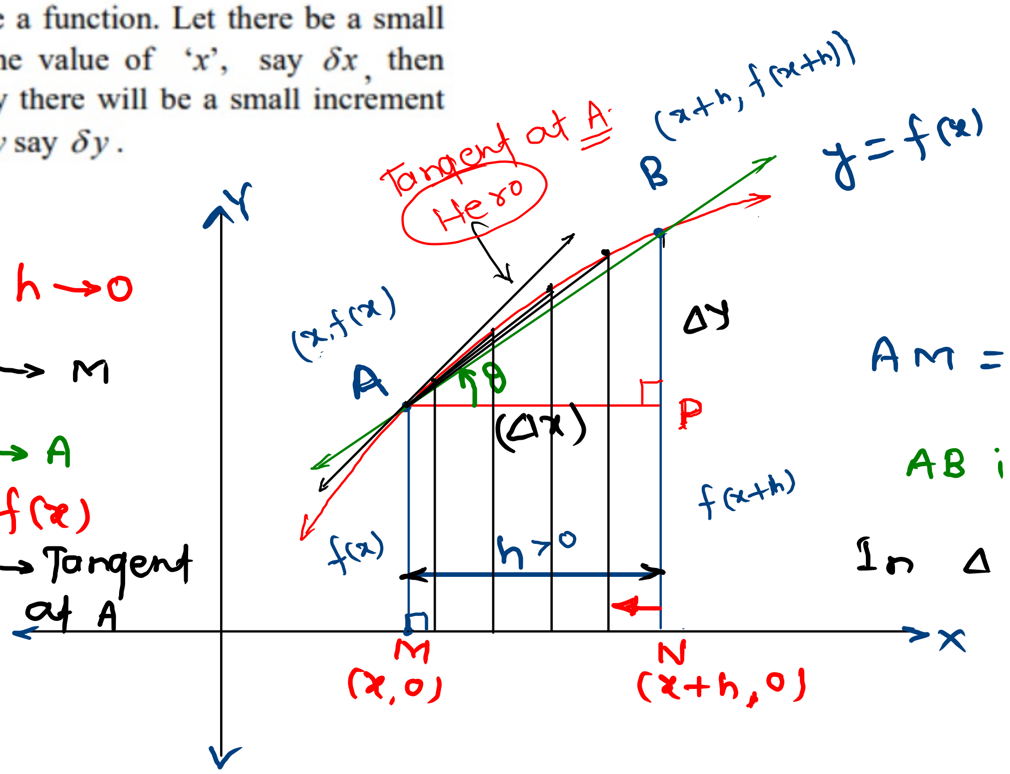
Consider $h \to 0$

$N \to M$

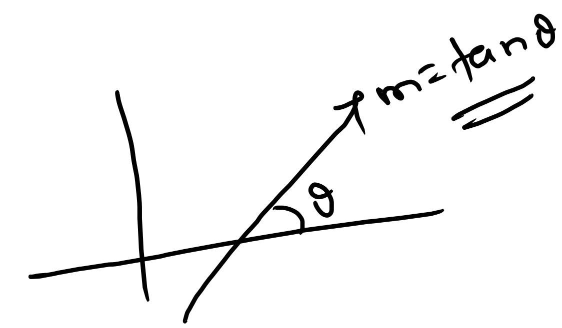
$B \to A$

$f(x+h) \to f(x)$

Secant $AB \to$ Tangent at A



AB is a secant



$AM = f(x)$, $BN = f(x+h)$

AB is a secant

In $\triangle APB$; $\tan\theta = \frac{BP}{AP}$ $\left\{ \begin{array}{l} h\to 0 \\ BP \to 0 \\ AP \to 0 \end{array} \right\}$ $\left\{ \begin{array}{l} BP = dy \\ AP = dx \end{array} \right\}$

$BP = BN - NP = BN - AM = f(x+h) - f(x)$

$\Rightarrow BP = f(x+h) - f(x)$

$AP = h$

$\tan\theta = \frac{f(x+h)-f(x)}{h}$;

Apply $\lim_{h\to 0}$ on B.S $\Rightarrow \lim_{h\to 0}(\tan\theta) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

This is called as derivative of $f(x)$ w.r.t x

and geometrically it gives the slope of tangent on the fun at $x=a$