## BOARD QUESTION PAPER : FEBRUARY 2020 MATHEMATICS AND STATISTICS

## Notes:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph paper is necessary for L.P.P
iv. Logarithm table will be provided on request.
v. Answers to the question in Section - I and Section - II should be written in two separate answer books.
vi. Question from Section - I attempted in the answer book of Section - II and vice-versa will not be assessed / not be given any credit.
vii. Write answer of every question on a separate page.

## Section - I

Q.1. Attempt any SIX of the following:
i. Write the negation of the following statements:
(a) If it snows, then Gajashri does not drive car.
(b) $\exists x \in \mathrm{~N}$, such that $x^{2}<x$.
ii. Let p: Tanmay is a student
$\mathrm{q}:$ Tanmay likes to watch cricket match
Write the verbal statement to describe each of the following:
(a) $p \leftrightarrow q$
(b) $\mathrm{p} \wedge \sim \mathrm{q}$
iii. Solve the following equations by the method of reduction:
$x+3 y=2,3 x+5 y=4$
iv. Find the value of $k$, if the function
$\mathrm{f}(x)=\frac{\tan 7 x}{2 x}$, for $x \neq 0$

$$
\begin{equation*}
=\mathrm{k} \quad, \text { for } x=0 \tag{2}
\end{equation*}
$$

is continuous at $x=0$
v. If $x=\log \left(1+\mathrm{t}^{2}\right)$ and $y=\log \mathrm{t}$, then find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
vi. Find the values of $x$ for which $\mathrm{f}(x)=3 x^{2}-15 x+9$ is decreasing.
vii. A triangle bounded by the lines $y=0, y=x$ and $x=4$ is revolved about X -axis. Find the volume of the solid of revolution using definite integral.
viii. Evaluate: $\int_{0}^{2} \frac{1}{\sqrt{4-x^{2}}} \mathrm{~d} x$
Q.2. (A) Attempt any TWO of the following:
i. Find the values of $x$ and $y$ from the equation:
$[-1,1,4]\left\{2\left[\begin{array}{cc}5 & 5 \\ 6 & 6 \\ -1 & 2\end{array}\right]+3\left[\begin{array}{cc}3 & 4 \\ 4 & 1 \\ 1 & -1\end{array}\right]\right\}=\left[\begin{array}{ll}x & y\end{array}\right]$
ii. Examine the continuity of the function:

$$
\begin{align*}
\mathrm{f}(x) & =x^{2}-x+9, & & \text { for } x \leq 3 \\
& =4 x+3, & & \text { for } x>3 \tag{3}
\end{align*}
$$

iii. If $x^{2} y^{\mathrm{k}}=(x+y)^{2+\mathrm{k}}$, then show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{y}{x}$

## (B) Attempt any TWO of the following:

i. Using the truth table, verify $\sim(\sim p \rightarrow \sim q) \equiv \sim p \wedge q$
ii. Find MPC, MPS, APC and APS, if the expenditure $E_{c}$ of a person with his income $I$ is given as $\mathrm{E}_{\mathrm{c}}=(0.0003) \mathrm{I}^{2}+(0.075) \mathrm{I}$, when $\mathrm{I}=1000$.
iii. Evaluate: $\int \frac{x^{2}}{x^{4}+5 x^{2}+6} \mathrm{~d} x$
Q.3. (A) Attempt any TWO of the following:
i. Find the adjoint of the matrix $\left[\begin{array}{ccc}1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1\end{array}\right]$
ii. If $y=x^{x}+(7 x-1)^{x}$, then find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
iii. Examine the continuity of the function
$\mathrm{f}(x)=\frac{5^{x}-3^{x}}{4^{x}-3^{x}}$, for $x \neq 0$

$$
\begin{equation*}
=\log \frac{5}{4}, \text { for } x=0 \tag{3}
\end{equation*}
$$

at $x=0$
(B) Attempt any TWO of the following:
i. A manufacturer can sell $x$ items at a price of $₹(280-x)$ each. The cost of producing $x$ items is $₹\left(x^{2}+40 x+35\right)$. Find the number of items to be sold so that the manufacturer can make maximum profit.
ii. Solve: $\int x^{2} \sin x d x$
iii. Evaluate : $\int_{3}^{9} \frac{\sqrt[3]{12-x}}{\sqrt[3]{x}+\sqrt[3]{12-x}} \mathrm{~d} x$

## Section - II

## Q.4. Attempt any SIX of the following:

i. An agent was paid ₹ 58,500 as commission on the sale of computers at the rate of $12.5 \%$. If the price of each computer was ₹ 18,000 , how many computers did he sell?
ii. Compute Age-specific death rate (Age-SDR) for the population from the following data:

| Age group <br> (Years) | Population |  |
| :---: | :---: | :---: |
|  | No. of <br> Persons | No. of <br> Deaths |
| $0-10$ | 600 | 18 |
| $10-25$ | 1,000 | 5 |
| $25-65$ | 3,000 | 24 |
| $65-100$ | 400 | 20 |

iii. From the following data with usual notations find correlation coefficient between X and Y :
$\mathrm{n}=50, \sum\left(x_{\mathrm{i}}-\bar{x}\right)\left(y_{\mathrm{i}}-\bar{y}\right)=420, \sigma_{x}=4, \sigma_{y}=3$.
iv. Determine the value of $k$ for the following probability distribution of X :

| $\mathrm{X}=x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | k | 2 k | 4 k | 2 k | k |

Also find $\mathrm{P}(\mathrm{X}<3)$.
v. Show the solution set for the following inequation $x+4 y \leq 0$ graphically.
vii. A fair coin is tossed 5 times. Find the probability of obtaining exactly 5 heads.
viii. For what amount should a cargo worth ₹ 25,350 be insured so that in the event of loss, its value as well as cost of insurance many be recovered, the rate of premium being $2.5 \%$.

## Q.5. (A) Attempt any TWO of the following:

i. Given $l_{26}=9046, l_{27}=8898$ and $T_{26}=36,000$. Find the values of $L_{26}, T_{27}$ and $e_{26}^{0}$.
ii. Following table gives the data of X and Y :

| X | $80-90$ | $90-100$ | $100-110$ | $110-120$ |
| :---: | :---: | :---: | :---: | :---: |
| $80-90$ | 3 | 2 | - | - |
| $90-100$ | 3 | 6 | 4 | 1 |
| $100-110$ | - | 2 | - | 2 |
| $110-120$ | - | 1 | - | 1 |

Find:
a. Marginal frequency distribution of X.
b. Marginal frequency distribution of Y.
c. Conditional frequency distribution of $X$ when $Y$ lies between 90-100.
iii. Find the present value of an annuity immediate of ₹ 18,000 per annum, for 3 years at $9 \%$ per annum, compounded annually.
[Given: $\left.(1.09)^{-3}=0.7722\right]$
(B) Attempt any TWO of the following:
i. You are given the following information about advertising expenditure and sales:

|  | Advertisement <br> expenditure <br> (₹ in lakhs) <br> X | Sales <br> (₹ in lakhs) <br> Y |
| :---: | :---: | :---: |
| Arithmetic <br> mean | 10 | 90 |
| Standard <br> deviation | 3 | 12 |

Correlation coefficient between X and Y is 0.8 .
a. Obtain the two regression equations.
b. What will be the likely sales when the advertising budget is ₹ 15 lakhs?
ii. A random variable X follows Poisson distribution such that -
$P[X=2]=\left(\frac{3}{4}\right) \cdot P[X=1]$. Find $P[X>1]$
[Use: $\mathrm{e}^{-15}=0.2231$ ]
iii. Five jobs are performed first on machine $M_{1}$ and then on machine $M_{2}$. Time taken in hours by each job on each machine is given below:

| Machines | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{M}_{1}$ | 6 | 8 | 4 | 5 | 7 |
| $\mathrm{M}_{2}$ | 3 | 7 | 6 | 4 | 16 |

Determine the optimal sequence of jobs and total elapsed time. Also find the idle time for machine $\mathrm{M}_{2}$.
Q.6. (A) Attempt any TWO of the following:
i. The equations of the two regression lines are $2 x+3 y-6=0$ and $5 x+7 y-12=0$. Find the correlation coefficient.
ii. From the following data compute CDR for town $A$ and town B. Interpret the result:

| Age group <br> (Years) | Town A |  | Town B |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Population <br> (in'000) | Number <br> of deaths | Population <br> (in '000) | Number of <br> deaths |
| $0-15$ | 10 | 200 | 14 | 320 |
| $15-60$ | 30 | 300 | 44 | 490 |
| 60 and above | 20 | 400 | 21 | 462 |

iii. 10 competitors in a beauty contest were ranked by two judges $A$ and $B$ as given below:

| Competitors | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank by A | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| Rank by B | 3 | 5 | 8 | 4 | 7 | 10 | 2 | 1 | 6 | 9 |

Compute rank correlation coefficient.
(B) Attempt any TWO of the following:
i. A computer centre has 4 expert programmers. The centre needs four application programmes to be developed. The head of the computer centre after studying the programmes to be developed, estimates the computer time (in hours) required by the respective experts to develop the application programme is as follows:

|  | Programmes |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| C | 10 | 25 | 2 | 14 |
| D | 15 | 7 | 14 | 10 |

How will the head of the computer centre assign the programmes to the programmers so that the total time (in hours) required is minimum?
ii. Solves the following L.P.P. graphically:

Minimize: $Z=3 x+2 y$,
Subject to the constraints

$$
\begin{align*}
& x-y \leq 1 \\
& x+y \geq 3 \\
& x \geq 0, y \geq 0 \tag{4}
\end{align*}
$$

iii. A bill of ₹ 4,000 drawn on $5^{\text {th }}$ January, 1998 for 8 months was discounted for ₹ 3,840 on a certain date. Find the date on which it was discounted at $10 \%$ per annum.

