DAY — **09** SEAT NUMBER 2 2 2 5 1 2

2023	III	03	1100	J-266	(E)

MATHEMATICS & STATISTICS (88)
(COMMERCE)

Time : 3 Hrs. **(12 Pages)** **Max. Marks : 80**

General Instructions :

(i) *All questions are compulsory.*

(ii) *There are 6 questions divided into two sections.*

(iii) *Write answers of Section-I and Section-II in the same answer book.*

(iv) *Use of logarithmic tables is allowed. Use of calculator is not allowed.*

(v) *For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.*

(vi) *Start answer to each question on a new page.*

(vii) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter eg.*

(a) ..../ (b) ..../ (c) ..../ (d) ....

No mark(s) shall be given if "ONLY" the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.

SECTION - I

Q. 1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]** (6)

(i) The dual statement of the statement $(p \vee q) \wedge (r \vee s)$ is ....

(a) $(p \wedge q) \wedge (r \wedge s)$ (b) $(p \wedge q) \vee (r \wedge s)$

(c) $(p \vee q) \vee (r \vee s)$ (d) $(r \vee s) \wedge (p \vee q)$

(ii) If $y = x \cdot \log x$ then $\frac{dy}{dx} = \text{.........}$

(a) 1 (b) $\frac{1}{x}$

(c) $\log x$ (d) $1 + \log x$

(iii) If $y = 2x^2 + a^2 + 2^2$ then $\frac{dy}{dx} = \text{.......}$

(a) $4x$ (b) $4x + 2a$

(c) $4x + 4$ (d) $2x$

(iv) A function f is said to be increasing at a point c if ....

(a) $f'(c) = 0$ (b) $f'(c) > 0$

(c) $f'(c) < 0$ (d) $f'(c) = 1$

(v) $\int_0^2 e^x dx = \text{......}$

(a) $e - 1$ (b) $1 - e$

(c) $e^2 - 1$ (d) $1 - e^2$

(vi) The integrating factor of $\frac{dy}{dx} + y = e^{-x}$ is ....

(a) e^x (b) e^{-x}

(c) x (d) $-x$

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) The derivative of $f(x)=a^x$ is $x \cdot a^{x-1}$ where a is constant.

(ii) $\int (1-x)^{-2}\,dx = (1-x)^{-1}+c$

(iii) The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = a^x \text{ is } 3.$$

(C) Fill in the following blanks (1 mark each) : (3)

(i) Converse of the statement $q \rightarrow p$ is ....

(ii) If $y=(\log x)^2$ then $\frac{dy}{dx}=$.......

(iii) If $0<\eta<1$ then the demand is ....

Q. 2. (A) Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Construct the truth table for the statement pattern $(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$

(ii) If $x^5 \cdot y^7 = (x+y)^{12}$ then show that $\frac{dy}{dx}=\frac{y}{x}$

(iii) Evaluate : $\int \frac{1}{7+6x-x^2}\,dx$

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Solve the following equations by reduction method :

$x+2y+z=8,\ 2x+3y-z=11,\ 3x-y-2z=5$

(ii) For manufacturing x units, labour cost is ₹(150-54x) and processing cost is ₹(x^2). Price of each unit is P = $10800-4x^2$. Find the values of x for which

(a) Total cost is decreasing

(b) Revenue is increasing

(iii) Evaluate : $\int_1^3 \frac{\sqrt{x+5}}{\sqrt{x+5}+\sqrt{9-x}}\,dx$

Q. 3. **(A)** Attempt any TWO of the following questions (3 marks each): (6) **[14]**

(i) Write the negation of each of the following statements :

(a) $\exists\, n \in N$, (n^2+2) is odd number.

(b) Some continuous functions are differentiable.

(c) $(p \to q) \vee (p \to r)$

(ii) A rod of 108 meters long is bent to form rectangle. Find its dimensions if the area of rectangle is maximum.

(iii) Evaluate : $\int x^3 \cdot \log x\, dx$

(B) Attempt any ONE of the following questions : (4)

(i) Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$

(ii) Find the particular solution of the differential equation $(x - y^2x)\,dx - (y + x^2y)\,dy = 0$ when $x = 2, y = 0$.

(C) Attempt any ONE of the following questions (Activity) : (4)

(i) If $A = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$

Find $(AB)^{-1}$ by adjoint method.

Solution :

$$AB = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$AB = \boxed{[\quad]}$$

$$|AB| = \boxed{\quad}$$

$M_{11} = -2$ $\quad\therefore\quad$ $A_{11} = (-1)^{1+1} . (-2) = -2$

$M_{12} = -3$ $\quad\quad$ $A_{12} = (-1)^{1+2} . (-3) = 3$

$M_{21} = 4$ $\quad\quad$ $A_{21} = (-1)^{2+1} . (4) = -4$

$M_{22} = 3$ $\quad\quad$ $A_{22} = (-1)^{2+2} . (3) = 3$

$$\text{Cofactor Matrix } [\text{Aij}] = \begin{bmatrix} -2 & 3 \\ -4 & 3 \end{bmatrix}$$

$$\text{adj (A)} = \boxed{[\quad]}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj (A)}$$

$$A^{-1} = \boxed{\quad}$$

(ii) In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, find the number of times the bacteria are increased in 12 hours.

Solution :

Let N be the number of bacteria present at time 't'. Since the rate of increase of N is proportional to N, the differential equation can be written as –

$$\frac{dN}{dt} \alpha\ N$$

$$\therefore \frac{dN}{dt} = KN$$

where K is constant of proportionality

$$\therefore \frac{dN}{N} = K \cdot dt$$

$$\therefore \int \frac{1}{N} dN = K \int 1 \cdot dt$$

$$\therefore \log N = \boxed{\quad} + C \quad \text{.........(1)}$$

when $t = 0$, $N = No$ where No is initial number of bacteria.

$$\therefore \log No = K \times 0 + C$$

$$\therefore C = \log No$$

Also when $t = 4$, $N = 2No$

$\therefore \log(2No) = K \cdot 4 + \boxed{}$ ....from (1)

$$\therefore \log\left(\frac{2No}{No}\right) = 4K,$$

$$\therefore \log 2 = 4K$$

$\therefore K = \boxed{}$ .... (2)

Now $N = ?$ when $t = 12$

from (1) and (2)

$$\log N = \frac{1}{4}\log 2 \cdot (12) + \log No$$

$$\log N - \log No = 3\log 2$$

$$\therefore \log\left(\frac{N}{No}\right) = \boxed{}$$

$$\therefore N = 8No$$

$\therefore$ Bacteria are increased 8 times in 12 hours.

SECTION - II

Q. 4. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]** (6)

(i) The sum due is also called as____.

(a) Face value (b) Present value

(c) Cash value (d) True discount

(ii) ____ is a series of cash flows over a limited period of time.

(a) Policy value (b) Annuity

(c) Present value (d) Future value

(iii) b_{yx} is ______.

(a) Regression coefficient of y on x

(b) Regression coefficient of x on y

(c) Correlation coefficient between x and y

(d) Covariance between x and y

(iv) The complicated but efficient method of measuring trend of time series is ______.

(a) graphical method

(b) method of moving average

(c) method of least squares

(d) method of addition

(v) Quantity index number by Weighted Aggregate Method is given by ______.

(a) $\sum \frac{q_1 w}{q_0 w} \times 100$ (b) $\sum \frac{q_0 w}{q_1 w} \times 100$

(c) $\frac{\sum q_0 w}{\sum q_1 w} \times 100$ (d) $\frac{\sum q_1 w}{\sum q_0 w} \times 100$

(vi) If the corner points of the feasible region are (0, 0), (3, 0), (2, 1) and (0, 7/3), then maximum value of $z = 4x + 5y$ is ______.

(a) 12 (b) 13

(c) $\frac{35}{3}$ (d) 0

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) The banker's discount is also called true discount.

(ii) Laspeyre's Price Index Number uses current year's quantities as weights.

(iii) In an assignment problem, if number of columns is greater than number of rows, then a dummy column is added.

(C) Fill in the blanks (1 mark each) : (3)

(i) The date by which the buyer is legally allowed to pay the amount is known as _____.

(ii) Walsch's Price Index Number is given by _______ .

(iii) Graphical solution set of the inequations $x \geq 0$ and $y \leq 0$ lies in __________ quadrant.

Q. 5. (A) Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) An agent places insurance for ₹ 4,00,000 on life of a person. The premium is to be paid annually at the rate of ₹35 per thousand per annum. Find the agent's commission at 15% on the premium.

(ii) For a bivariate data :

$$\sum(x-\bar{x})^2 = 1200, \sum(y-\bar{y})^2 = 300$$

$$\sum(x-\bar{x})(y-\bar{y}) = -250$$

Find : (a) b_{yx} (b) b_{xy} (c) Correlation coefficient between x and y.

(iii) The following table shows gross capital information (in Crore ₹) for years 1966 to 1975 :

Years	**1966**	**1967**	**1968**	**1969**	**1970**
Gross Capital information	20	25	25	30	35
Years	**1971**	**1972**	**1973**	**1974**	**1975**
Gross Capital information	30	45	40	55	65

Obtain trend values using 5-yearly moving values.

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Solve the following LPP by graphical method :

Maximize : $z = 4x + 6y$

Subject to $3x + 2y \leq 12, x + y \geq 4,$

$x \geq 0, y \geq 0.$

(ii) A marketing manager has list of salesmen and territories. Considering the travelling cost of the salesmen and the nature of the territory, the marketing manager estimates the total of cost per month (in thousand rupees) for each salesman in each territory. Suppose these amounts are as follows :

Salesman	**Territories**				
	I	**II**	**III**	**IV**	**V**
A	11	16	18	15	15
B	7	19	11	13	17
C	9	6	14	14	7
D	13	12	17	11	13

Find the assignment of salesman to territories that will result in minimum cost.

(iii) A random variable X has the following probability distribution :

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find : (a) k (b) $P(X < 3)$ (c) $P(X > 4)$

Q. 6. **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) An agent was paid ₹ 88,000 as commission on the sales of computers at the rate of 12.5%. If the price of each computer was ₹ 32,000, how many computers did he sell?

(ii) The publisher of a magazine wants to determine the rate of increase in the number of subscribers. The following table shows the subscription information for eight consecutive years :

Years	1976	1977	1978	1979
No. of subscribers (in millions)	12	11	19	17
Years	**1980**	**1981**	**1982**	**1983**
No. of subscribers (in millions)	19	18	20	23

Fit a trend line by graphical method.

(iii) If x follows Poisson distribution such that $P(x = 1) = 0.4$ and $P(x = 2) = 0.2$, find variance of x.

(B) Attempt any ONE of the following questions : (4)

(i) From the following data, find the regression equation of y on x and estimate y when $x = 4$:

x	1	2	3
y	2	1	6

(ii) Calculate Marshall – Edgeworth's price index number for the following data :

Commodity	Base year		Current year	
	Price	Quantity	Price	Quantity
P	12	20	18	24
Q	14	12	21	16
R	8	10	12	18
S	16	15	20	25

$2x-y+3-y$

$2x-1-y$

(C) Attempt any ONE of the following questions (Activity) : (4)

(i) Six jobs are performed on Machines M_1 and M_2 respectively. Time in hours taken by each job on each machine is given below :

Jobs → / Machines ↓	A	B	C	D	E	F
M_1	3	12	5	2	9	11
M_2	8	10	9	6	3	1

Determine the optimal sequence of jobs and find total elapsed time. Also find the idle time for machines M_1 and M_2.

Solution :

Given jobs can be arranged in optimal sequence as,

D	A	C	B	E	F

Jobs	Machine M_1		Machine M_2	
	In	Out	In	Out
D	0	2	$\square$	8
A	2	5	8	16
C	5	10	16	25
B	10	22	25	35
E	22	31	35	38
F	31	42	$\square$	43

Total Elapsed time = $\square$ hrs.

Idle time for Machine M_1 = 43 – 42 = 1 hour.

Idle time for Machine M_2 = $\square$ hrs.

(ii) A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of getting at least two success.

Solution :

A pair of dice is thrown 3 times

$\therefore \quad n = 3$

Let x = number of success (doublets)

p = probability of success (doublets)

$\therefore \quad p = \boxed{}, \quad q = \boxed{}$

$\therefore \quad x \sim B(n, p)$

$$P(x) = {}^{n}C_{x} p^{x} q^{n-x}$$

Probability of getting at least two success means $x \geq 2$.

$$\therefore \quad P(x \geq 2) = P(x = 2) + P(x = 3)$$

$$= \boxed{} + \boxed{}$$

$$= \frac{2}{27}$$

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